Bayesian Computation

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In Bayesian statistics the posterior distribution $p(\psi|\, y)$ contains all relevant information on the unknown parameters ψ given the observed data y. All statistical inference can be deduced from the posterior distribution by reporting appropriate summaries. This typically takes the form of evaluating integrals $J = \int f(\psi)\ p(\psi|\, y)\ d\psi$ of some function $f(\psi)$ with respect to the posterior distribution. The problem is that these integrals are usually impossible to evaluate analytically. And when the parameter is multidimensional, even numerical methods may fail. Over the last fifteen years a barrage of literature has appeared concerned with the evaluation of such integrals by methods collectively known as Monte Carlo and Markov chain Monte Carlo (MCMC) simulation. Using MCMC simulation it is possible to implement posterior integration in essentially any problem which allow pointwise evaluation of the prior distribution and likelihood function.

In this tutorial, we will review the most popular MC(MC) algorithms. We will discuss importance sampling, data augmentation, Gibbs sampling, and Metropolis-Hastings algorithms. The underlying rationale of Monte Carlo integration is the use of computer simulation to generate a sample from the posterior distribution. Sample averages over such Monte Carlo samples approximate the posterior integrals of interest. MCMC algorithms simulate the posterior sample by setting up a Markov chain that is constructed to have the desired posterior distribution as its limiting distribution. The art of MCMC is to set up a suitable Markov chain with the given posterior as stationary distribution and to judge when to stop simulation, i.e, to diagnose when the chain has practically converged. We will discuss related results and commonly used convergence diagnostics.

If time permits we will conclude by reviewing Monte Carlo simulation in dynamic state space models, a popular framework for Bayesian inference for time series data. Monte Carlo algorithms in such models are known as particle filters.

Below is a (somewhat arbitrary) selection of introductory texts and papers related to Bayesian computation. Gamerman's (1997) book covers most of the topics discussed in this tutorial at an easily accessible level of mathematical detail.

Gamerman, D., (1997), *Markov Chain Monte Carlo*, London: Chapman and Hall.

Gelfand, A. and Smith, A., (1990), "Sampling based approaches to calculating marginal densities," *Journal of the American Statistical Association*, 85, 398–409.

Gilks, W., Richardson, S., and Spiegelhalter, D., (1996), *Markov chain Monte Carlo in practice*, Chapman and Hall.

Robert, C. and Casella, G., (1999), *Monte Carlo Statistical Methods*, Springer-Verlag.

Tanner, M., (1996), *Tools for Statistical Inference: Methods for the Exploration of Posterior Distributions and Likelihood Functions*, 3rd. ed., New York: Springer-Verlag.

Tierney, L., (1994), "Markov chains for exploring posterior distributions," *Annals of Statistics*, 22, 1701–1728.